DETERMINATION OF THE CALIBRATION CURVE IN BRIDGMAN ANVILS

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In carrying out experiments or in putting into practice one or another industrial process in the high-pressure region, we need a rather accurate knowledge of the value of the stresses at the location of the substance being studied or processed. The functional connection between the above stress and the directly determined value, i.e., the total force applied to the unit, is called the calibration curve. The reliability of our knowledge of the calibration curve also determines the reliability of the operation of any given high-pressure unit, since direct measurements of the pressure are practically excluded.

In high-pressure units, which use a liquid as a working medium, plotting of the calibration curve is rather simple. It is a different matter in units which use a plastic substance as a working medium (with the present-day level of experimental techniques in the field of high-pressure physics, it is precisely these units which permit obtaining maximal pressures). The present article considers this question using the example of a rather characteristic unit, i.e., a Bridgman anvil.

A Bridgman anvil may be represented schematically in the form of two elastic half-spaces, with a thin disc made of a plastic material arranged between them. With approach of the half-spaces, the disc becomes thinner and its radius increases. Due to the presence of friction between the plastic medium and the half-spaces (there is no lubricant), there arises a pressure gradient, increasing with a decrease in the thickness of the disc. To simplify the analysis, we neglect the effect of the foreign inclusion located in the plastic medium (the substance being investigated) since, as a rule, inclusions of this type are relatively small. We also assume that the working medium is an ideally plastic body (σ_S is the yield point).

Thus, the problem of determining the contact pressures in Bridgman anvils can be formulated as the problem of the axisymmetric flow of a thin layer of plastic material over deformed surfaces [1].

We introduce the polar system of coordinates r, φ; $p(r)$, $\tau(r)$, and $h(r)$ denote, respectively, the contact pressure, the friction stress, and the half-thickness of the plastic disc. We denote its radius by R, and the half-thickness at the edge, i.e., at $r = R$, by H.

Since $h(r) \ll R$, in accordance with [1], we have the equation

$$\frac{dp}{dr} = \frac{\tau(r)}{h(r)} \tag{1}$$

The friction stress $\tau(r)$ is assumed to be a given function. The contact pressure at the edge is taken equal to

$$p(R) = 0.5\sigma_s\,(1 + \delta + \cos\delta) \quad (\delta = \arcsin \tau(R)/0.5\sigma_s)$$

The above value of $p(R)$ is determined from the exact solution of the equations of plastic equilibrium in the neighborhood of the edge [2]. If the friction coefficient is equal to $0.5\ \sigma_S$, then, $p(R) = 1.28\ \sigma_S$.

To determine the displacement of the surface of the half-spaces, $w(r)$, we use the Boussinesq formula (see, for example, [3]) which, with application to the axisymmetric case is written as follows:

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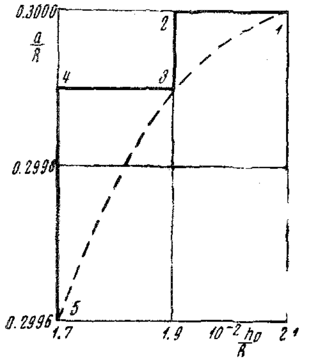

Fig. 1

$$w(r)=\frac{1-\mu^2}{\pi E}\int_0^R\int_0^{2\pi}\frac{p(r')\,r'\,d\varphi\,dr'}{\sqrt{(r'\cos\varphi-r)^2+(r'\sin\varphi)^2}} \tag{2}$$

where μ and E are the Poisson coefficient and the elastic modulus. The following equality is obvious:

$$h(r)=w(r)-w(R)+H \tag{3}$$

The problem consists of solving the system of equations (1), (2) simultaneously with equality (3), with a given total force and volume of the plastic medium. However, from a mathematical point of view, it is expedient to assume that the dimensions R and H are given. Constructing the solution, we find the sought force and volume. The final result can be represented in the form of a calibration curve.

In [4] an investigation was made of the system (1), (2) under the condition $\tau(r) = -0.5\,\sigma_S$. From the postulation of a constant sign of the friction stresses, on the basis of Eq. (1) it follows immediately that in absolute value the pressure rises from the edge toward the center, attaining its maximal value there. However, the experimental investigations of [5] and others have shown that, with the compression of very thin discs, the pressure in the central part becomes non-maximal.

In [6] an attempt was made to explain this effect. The author took the road of a rather arbitrary division of the plastic medium into an outer ring of plastic material, and a region located within, filled with a compressible liquid. The result of the proposed model was a change in the sign of the friction stresses on part of the contact surface.

We pass on to an explanation of the proposed model. We assume that at each moment of the deformation there exists a circle (we call it neutral, and denote its radius by a) which has the property that the plastic material located within it, during the course of the deformation, is displaced toward the center, while that located outside the circle is displaced toward the periphery. Therefore, the friction stress $\tau(r)$ must be positive with $r < a$, and negative with $r > a$. In the calculating scheme, for purposes of simplicity, we shall assume that the friction stress is constant in value and equal to $0.5\,\sigma_S$.

The system of equations (1), (2) can be solved for any arbitrary value of the radius a, with given values of H and R. To determine the actual value of the radius a, at fixed values of H and R, we must bring in supplementary considerations. Furthermore, for purposes of simplification, in addition to taking a more accurate account of the form of the working surfaces of the Bridgman anvil (the punches have the form of truncated cones with small angles of conicity), we assume that the material flowing out beyond the limits of the working surfaces has no effect on the further course of the process. We assume also that the working volume has already been filled; therefore, with a further approach of the punches, there is no change in the value of R.

We assume that the system (1), (2) is solved for given values of a, H, and R, under the above conditions (the method of solution is expounded below). We introduce into the consideration the function $\Omega(r, H, a)$, equal to the volume of plastic material included within a circle of radius r. In this case, if we take account of the compressibility of the plastic material, the function $\Omega(r, H, a)$ must determine not the volume, but the corresponding mass.

Before passing on to the establishment of a connection between the quantities a and H, let us consider the question of the dynamics of the neutral circle. Let there exist some solution, determined by the set of values of R, H_1, and a_1. We assign to the quantity H_1 a certain increment ΔH_1.

How shall we determine the corresponding increment of a ? Assuming that the value of ΔH_1 is small in comparison with H_1, on the basis of the determined concept of the neutral circle, we can write the equality

$$\Omega(a_1+\Delta a_1,\ H_1+\Delta H_1,\ a_1)=\Omega(a_1,\ H_1,\ a_1) \tag{4}$$

With an accuracy up to infinitesimals of higher order, equality (4) may be represented in the form

$$\Delta a_1\left.\frac{\partial\Omega(r,\ H_1,\ a_1)}{\partial r}\right|_{r=a_1}+\Delta H_1\frac{\partial\Omega(a_1,\ H_1,\ a_1)}{\partial H_1}=0$$

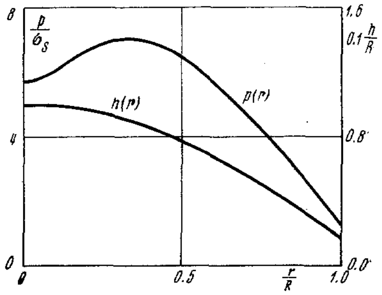

Fig. 2

Passing to the limit, we arrive at the differential equation

$$\frac{da}{\partial H} = -\frac{\partial\Omega(a, H, a)}{\partial H}\left(\frac{\partial\Omega(r, H, a)}{\partial r}\bigg|_{r=a}\right)^{-1} \tag{5}$$

Equation (5), generally speaking, permits determining a dependence between a and H, in the presence of noncontradictory starting data. For interpretations we turn to a comparison between solutions determined by the parameters a_1, H_1 and $a_1 + \Delta a_1$, $H_1 + \Delta H_1$. Let ΔH_1 be a negative quantity. If, with a decrease in the value of H, there is not an increase in the thickness at the center, then, the set of values of a_1 and H_1 must naturally be rejected. The limitation established may also be written in differential form.

Since the function $\Omega(r, H, a)$ cannot be determined in analytical form, the following procedure is proposed for the approximate construction of the dependence $a(H)$. On the curve of Fig. 1, we plot the point 1 with the coordinates H_1 and a_1. We assign to H_1 the increment ΔH_1. We construct the solution for the parameters $H_1 - \Delta H_1$ and a, and, by trial-and-error, we determine the value of r (denoted by a_3) for which the following equality is satisfied

$$\Omega(a_3, H_1 - \Delta H_1, a_1) = \Omega(a_1, H_1, a_1)$$

The point with the coordinates $H_1 - \Delta H_1$ and a_1, we denote by 2, and with the coordinates $H_1 - \Delta H_1$ and a_3, by 3. We further construct a solution for the parameters $H_1 - \Delta H_1 - \Delta H_3$ and a_3. We denote the corresponding point by 4, etc. The smooth dotted curve, connecting the points with uneven indices, approximately determines the required dependence.

We give below the results of calculations with the following values of the starting data: $\nu = 0.3$, $\sigma_s/E = 0.02$, $a/R = 0.3$, $H_1/R = 0.02034$, $\Delta H_1/R = -0.00154$, $\Delta H_3/R = -0.0012$.

	Point 1	Point 3	Point 5
H	0.02034	0.0188	0.0170
$h(0.5R)$	0.09321	0.931	0.0932
$h(0)$	0.11255	0.1128	0.1133
a/R	0.3000	0.2999	0.2996
Ω/R^3	0.4244	0.4209	0.4175
$P/\sigma_s R^2$	14.22	14.52	14.91

Here P denotes the total force and Ω the volume of the plastic medium.

With a transition from point 1 to points 3 and 5, there is a consecutive increase in the height at the center, of the total force, and a decrease of the volume of material, as the result of its flowing out beyond the working volume.

Figure 2 gives curves for p(r) and h(r) with the parameters $a = 0.3$ and $H_1/R = 0.02034$.

Let us continue our investigation of the question of determining the initial values of a and H. We return to the case of the compression of an originally rather thick disc. Naturally, all the material will be displaced away from the center, and the proposed scheme does not hold. In this case the value of H, at which flow toward the center arises is determined from the equation

$$\partial\Omega(0, H, 0)/\partial H = 0 \tag{6}$$

Establishment of the corresponding value of H is of great practical importance since, in this case, the region of actually constant pressure is greatly extended. Therefore, in carrying out experimental investigations, it is possible to displace a relatively large volume of the substance being investigated; the results, being averaged, are not distorted by the nonhomogeneity of the stress field.

If an originally hollow disc is compressed, the investigation must be carried out in accordance with [1], right up to the moment when the internal cavity degenerates. The value of the radius of the neutral circle is taken from the solution for a hollow disc at the moment when the cavity degenerates.

The question under consideration is considerably more complex with the compression of originally thin discs. For example, the following situation may arise. During the course of the compression, the outer part of the disc may go over into a plastic state and, in the central part of the elastic zone, there may arise breakaway of the material of the discs from the punches.

The forming cavity may be the reason for plastic flow toward the center. To determine the sought values of a and H, it is found necessary to follow all the stages of the solution described above, right up to the moment when the elastic zones degenerate.

The above considerations may be found to be extremely natural, if account is taken of the usual shaping of the working surfaces.

The above analysis shows that the concept of a "calibration curve" for high-pressure units, using a plastic body as a working medium, has meaning only with fixed starting dimensions of the plastic body. Attention must also be called to the fact that the calibration curve may be found not to be a mutually single-valued function.

Let us outline the calculating scheme. Let us consider the integral (2). We divide up the region of integration (a circle of radius R) by the set of l concentric, equally spaced circles, and by a bundle of m straight lines, passing through the center, into small curvilinear rectangles. We shall determine all the sought functions p(r) and h(r) at points with the radial coordinate $r_i = (i-0.5)\,R/l$, located at the centers of these rectangles. The subscript i takes on values from 1 to l. The boundary conditions for the functions p(r) and h(r) are taken at a point with the coordinate r.

Taking account of the presence in the kernel under the integral sign of a weak (integrable) singularity, we carry out the transformation

$$w(r)=\frac{1-\mu^2}{\pi E}\int_0^R\int_0^{2\pi}\frac{[p(r')-p(r)]\,r'\,dr'\,d\varphi}{\sqrt{(r'\cos\varphi-r)^2+(r'\sin\varphi)^2}}+\frac{1-\mu^2}{\pi E}\,p(r)\int_0^R\int_0^{2\pi}\frac{r'\,dr'\,d\varphi}{\sqrt{(r'\cos\varphi-r)^2+(r'\sin\varphi)^2}}$$

The kernel of the expression under the integral sign in the first integral is a continuous function; the second integral is taken in closed form [3]. It is found to be equal to

$$2\pi R\left[1-\sum_{n=1}^{\infty}\left(\frac{r}{R}\right)^{2n}\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)\cdot1\cdot3\cdot5\cdot\ldots\cdot(2n-3)}{(2\cdot4\cdot6\cdot\ldots\cdot2n)^2}\right]$$

The elementary sums are calculated as the product of the difference $p(r')-p(r)$, taken at the centers of the corresponding rectangles, and the mean value at the apexes of the rectangles, of the following expression

$$\frac{r'R\pi}{lm}\,\frac{1}{\sqrt{(r'\cos\varphi-r)^2+(r'\sin\varphi)^2}}$$

We return again to Eq. (1) and represent it approximately in the form

$$p(r_i)=1.28\sigma_s+\sum_{j=i}^{l-1}\frac{\tau(r_j)}{h(r_j)}\Delta r\qquad(i=1,2,\ldots,l-1)\tag{7}$$

The system of equations obtained is solved by the method of successive approximations. As a zero approximation, we take any given distribution of the pressures $p(r_i)$; we determine the corresponding values of $w(r_i)$ and, consequently, $h(r_i)$. From formula (7) we find the first approximation for the pressures $p(r_i)$; we repeat the proposed algorithm until a satisfactory convergence is achieved.

In the calculations, the results of which were presented earlier, as a zero approximation we took $p(r_i) = 1.28\,\sigma_S$, $l = 80$, m = 40, iteration number = 10. Under these circumstances, a degree of accuracy up to 5 digits was attained.

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